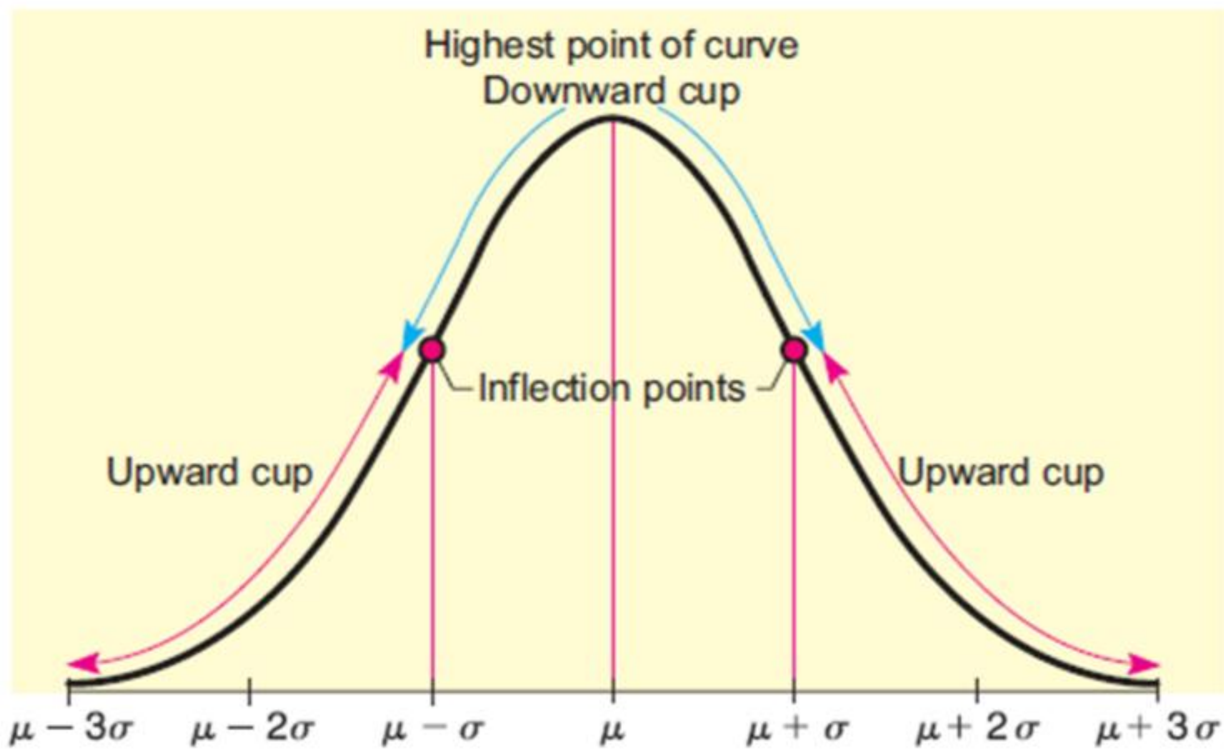
6.1 Graphs of Normal Probability Distributions

__________________________ – one of the most important examples of a continuous probability distribution, studied by Abraham de Moivre (1667 – 1754) and Carl Friedrich Gauss (1777 – 1855). (Sometimes called the ____________________ distribution.)

We could look at a very complicated formula which speaks of the normal distribution; however, we will just look at the graph of a normal distribution to get a better idea of what we are discussing.

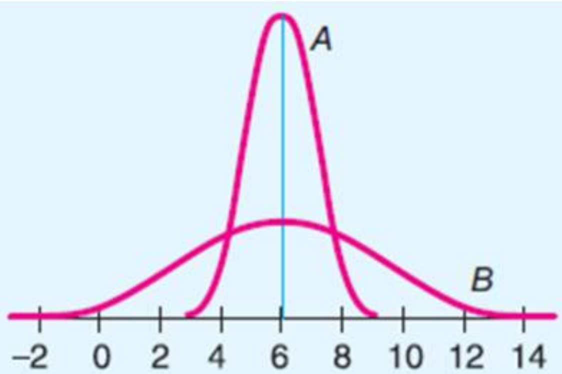
Graph of a Normal Distribution



Important properties of a normal curve

1. The curve is bell-shaped, with the highest point over the mean μ.
2. The curve is symmetrical about a vertical line through μ.
3. The curve approaches the horizontal axis but never touches or crosses it.
4. The inflection (transition) points between cupping upward and downward occur above $\mu + \sigma$ and $\mu - \sigma$.

Examples:



(a) Do these distributions have the same mean? If so, what is it?

(b) One of the curves corresponds to a normal distribution with $\sigma = 3$ and the other to one with $\sigma = 1$. Which curve has which σ?

Some Facts to Realize About Normal Curves:

1) The mean and standard deviation have ______________________ on each other. So, a curve with a large mean need not have a large standard deviation.

2) If a curve is very spread out, it then has a large standard deviation, and vice versa.

Examples:

Sketch the following curves given the information below. Label everything, including transition points:

a) Mean of 24 and Standard Deviation of 11

b) Mean of 19 and Standard Deviation of 6.

c) Mean of 111 and Standard Deviation of 10.

d) Mean of 107.5 and Standard Deviation of 9.

e) Mean of 20 and Standard Deviation of 6.2.

Empirical rule

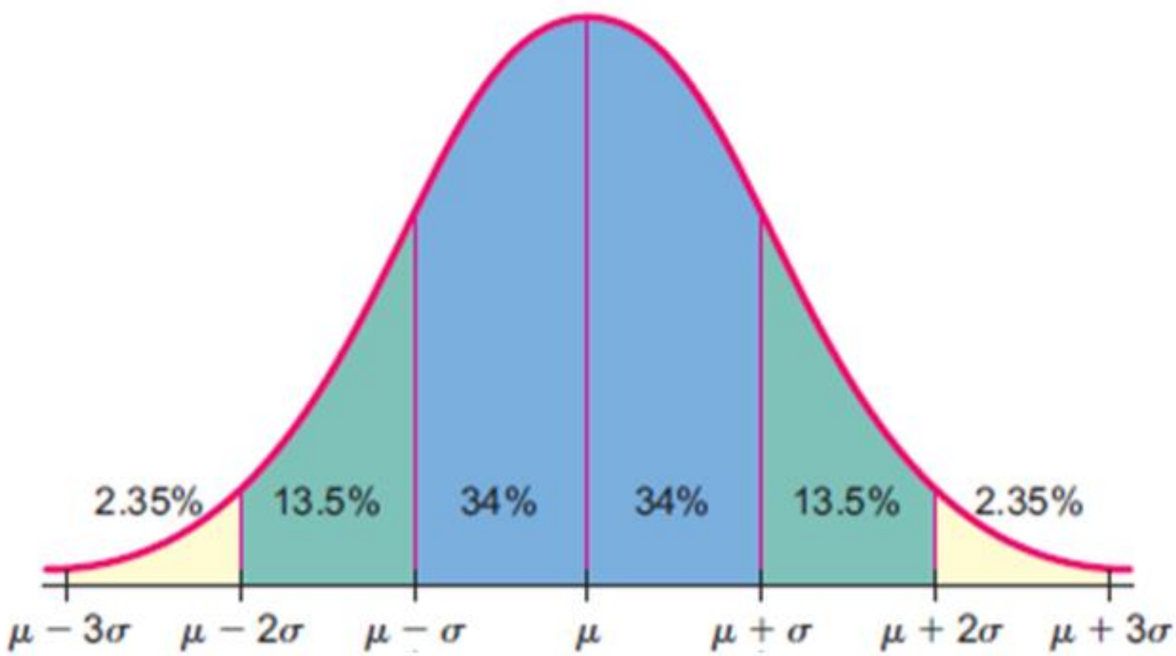
For a distribution that is symmetrical and bell-shaped (in particular, for a normal distribution):

Approximately 68% of the data values will lie within one standard deviation on each side of the mean.

Approximately 95% of the data values will lie within two standard deviations on each side of the mean.

Approximately 99.7% (or almost all) of the data values will lie within three standard deviations on each side of the mean.

Area Under a Normal Curve



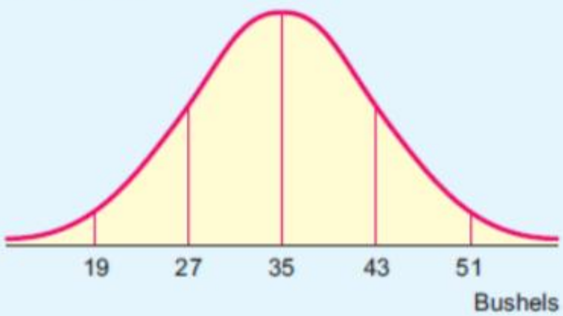
Examples:

The yearly wheat yield per acre on a particular farm is normally distributed with mean $\mu = 35$ bushels and standard deviation $\sigma = 8$ bushels.

(a) Shade the area under the curve in Figure 6-5 that represents the probability that an acre will yield between 19 and 35 bushels.

(b) Is the area the same as the area between $\mu - 2\sigma$ and μ?

FIGURE 6-5



(c) Use Figure 6-3 to find the percentage of area over the interval between 19 and 35.

(d) What is the probability that the yield will be between 19 and 35 bushels per acre?

Control Charts

If we are examining data over a period of equally spaced time intervals or in some sequential order, then *control charts* are especially useful. Business managers and people in charge of production processes are aware that there exists an inherent amount of variability in any sequential set of data. For example, the sugar content of bottled drinks taken sequentially off a production line, the extent of clerical errors in a bank from day to day, advertising expenses from month to month, or even the number of new customers from year to year are examples of sequential data. There is a certain amount of variability in each.

A random variable x is said to be in *statistical control* if it can be described by the *same* probability distribution when it is observed at successive points in time. Control charts combine graphic and numerical descriptions of data with probability distributions.

HOW TO MAKE A CONTROL CHART FOR THE RANDOM VARIABLE x

A control chart for a random variable x is a plot of observed x values in time sequence order.

1. Find the mean μ and standard deviation σ of the x distribution by
   (a) using past data from a period during which the process was "in control" or
   (b) using specified "target" values for μ and σ.
2. Create a graph in which the vertical axis represents x values and the horizontal axis represents time.
3. Draw a horizontal line at height μ and horizontal, dashed control-limit lines at $\mu \pm 2\sigma$ and $\mu \pm 3\sigma$.
4. Plot the variable x on the graph in time sequence order. Use line segments to connect the points in time sequence order.

Every 15 days Ms. Tamara has a general staff meeting at which she shows a control chart of the number of rooms not made up by 3:30 P.M. each day. From extensive experience, Ms. Tamara is aware that the distribution of rooms not made up by 3:30 P.M. is approximately normal, with mean $\mu = 19.3$ rooms and standard deviation $\sigma = 4.7$ rooms. This distribution of x values is acceptable to the top administration of Antlers Lodge. For the past 15 days, the housekeeping unit has reported the number of rooms not ready by 3:30 P.M. (Table 6-1). Make a control chart for these data.

TABLE 6-1 **Number of Rooms x Not Made Up by 3:30 P.M.**

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
x	11	20	25	23	16	19	8	25	17	20	23	29	18	14	10

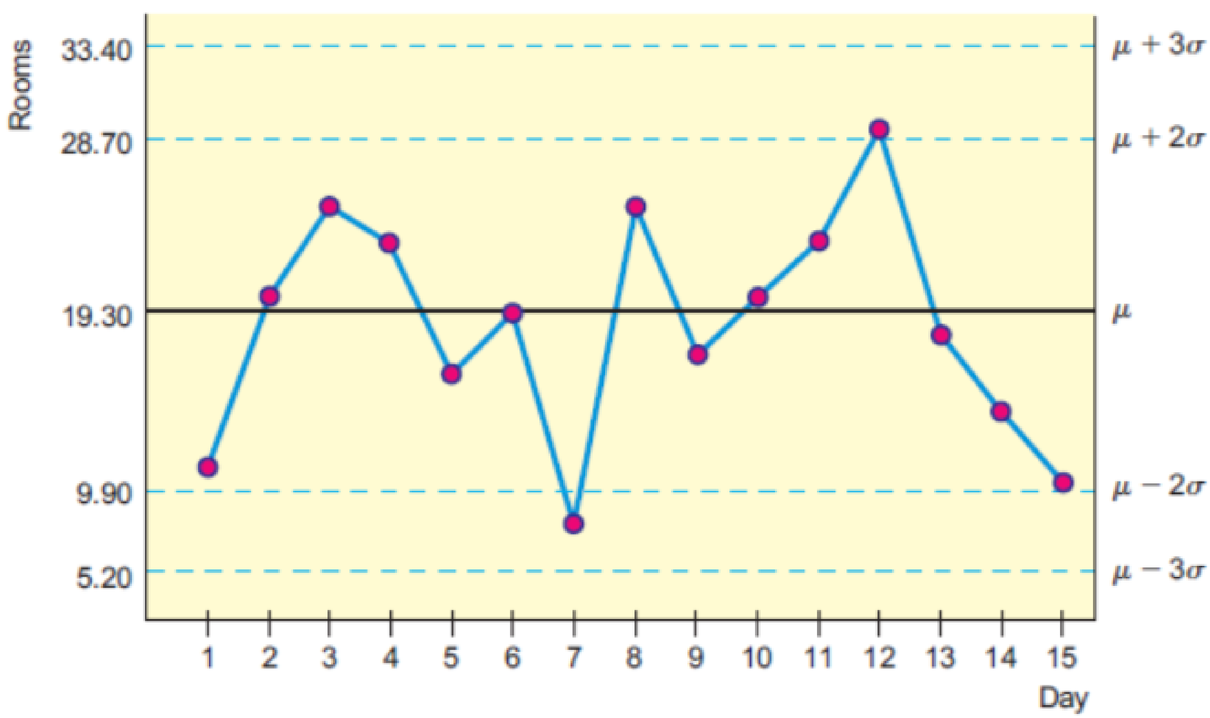
SOLUTION: A control chart for a variable x is a plot of the observed x values (vertical scale) in time sequence order (the horizontal scale represents time). Place horizontal lines at

the mean $\mu = 19.3$

the control limits $\mu \pm 2\sigma = 19.3 \pm 2(4.7)$, or 9.90 and 28.70

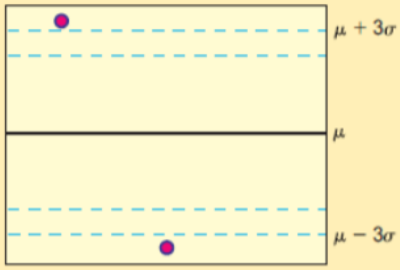
the control limits $\mu \pm 3\sigma = 19.3 \pm 3(4.7)$, or 5.20 and 33.40

Then plot the data from Table 6-1.



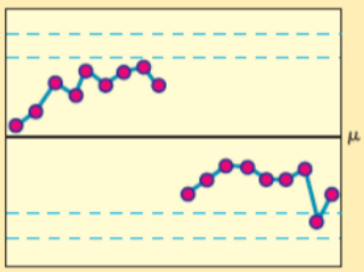
Out-of-control signals

Out-of-Control Signal I



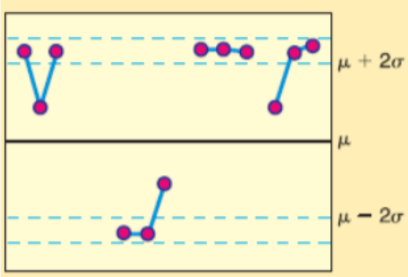
1. Out-of-Control Signal I: One point falls beyond the 3σ level

Out-of-Control Signal II



2. Out-of-Control Signal II: A run of nine consecutive points on one side of the center line (the line at target value μ)

Out-of-Control Signal III



3. Out-of-Control Signal III: At least two of three consecutive points lie beyond the 2σ level on the same side of the center line

Examples:

Figures 6-8 and 6-9 show control charts of housekeeping reports for two other 15-day periods.

FIGURE 6-8 Report II

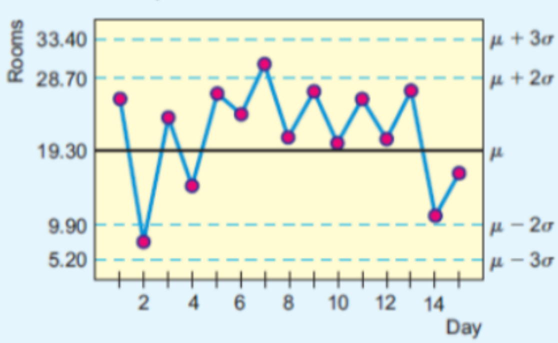
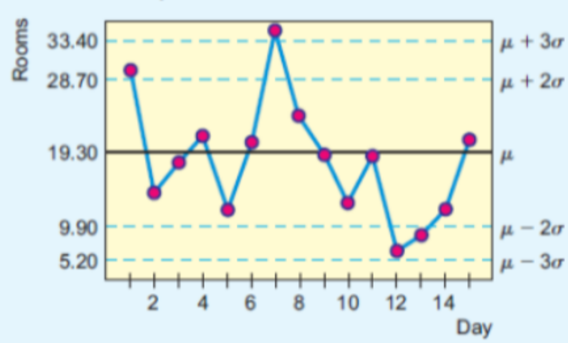


FIGURE 6-9 Report III



(a) Interpret the control chart in Figure 6-8.

(b) Interpret the control chart in Figure 6-9.

6.1 Homework

1) μ = 78   σ = 3

2) μ = 19   σ = 4.6

3) What percent of the data lies within 1 standard deviation of a normal distribution?

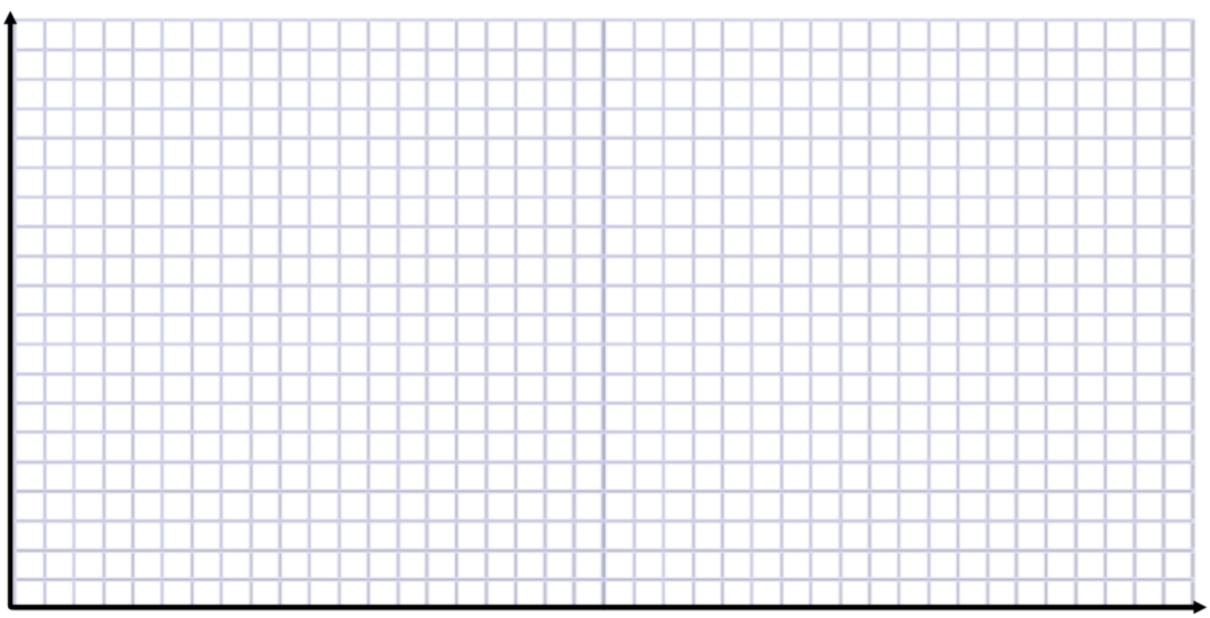
4) What percent of the data lies within 2 standard deviations of a normal distribution?

5) What percent of the data lies within 3 standard deviations of a normal distribution?

6) Create a Control Chart for the following data.

$\mu = 44$ $\sigma = 6$

1	2	3	4	5	6	7	8	9	10
50	36	28	45	40	48	51	46	33	48

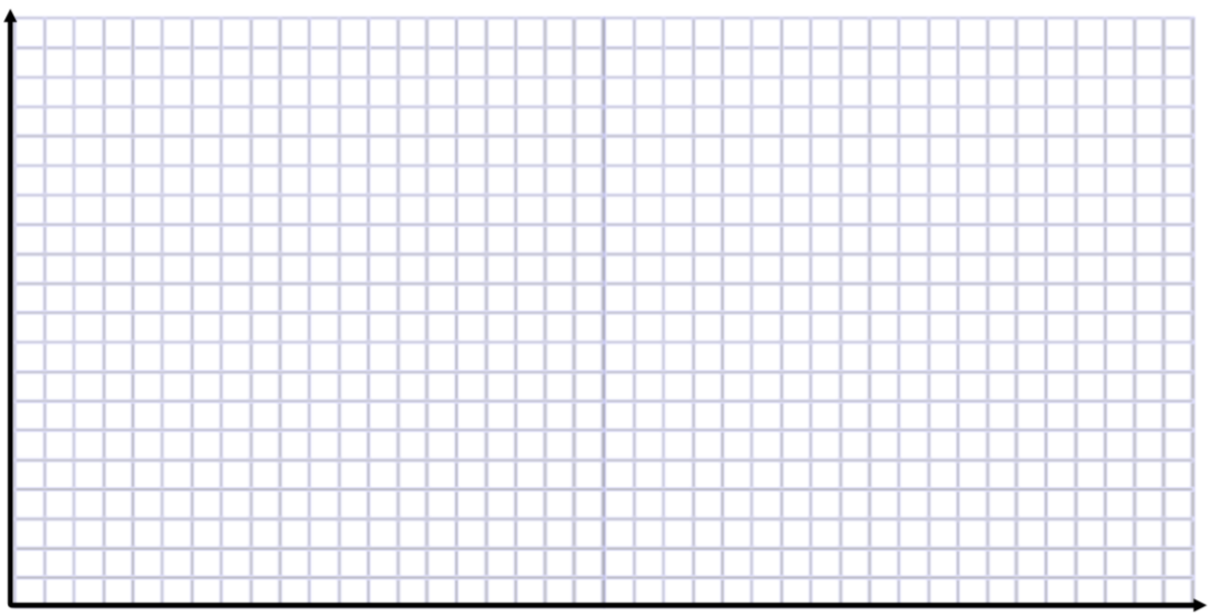


Is the above data in control? Why or why not?

7) Create a Control Chart for the following data.

$\mu = 82$ $\sigma = 4$

1	2	3	4	5	6	7	8	9	10
90	71	80	85	83	90	77	75	73	70



Is the above data in control? Why or why not?

6.2 Standard Units and Areas Under the Standard Normal Distribution

There is a simple formula that we can use to compute the number z of standard deviations between a measurement x and the mean μ of a normal distribution with standard deviation σ:

Definition: The z value or ____________________ tells us the number of standard deviations the original measurement is from the mean. The z value is in standard units.

The mean of the original distribution is always zero, in standard units, which makes sense because the mean is zero standard deviations from itself.

An x value in the original distribution that is above the mean μ has a corresponding z value that is __________________. Again, this makes sense because a measurement above the mean would be a positive number of standard deviations from the mean. Likewise, an x value below the mean has a ____________ z value. (See below!)

***x* Values and Corresponding *z* Values**

x Value in Original Distribution	Corresponding *z* Value or Standard Unit
$x = \mu$	$z = 0$
$x > \mu$	$z > 0$
$x < \mu$	$z < 0$

Examples:

1) Lewis earned 85 on his biology midterm and 81 on his history midterm. However, in the biology class the mean score was 79 with standard deviation 5. In the history class the mean score was 76 with standard deviation 3.

a) Convert the biology score to a standard score.

b) Convert the history score to a standard score.

c) Which score was higher with respect to the rest of the class?

2) Bill earned an 88 on his math final exam and an 85 on his history final exam. The mean score in the math class was 85 and the mean score in the history class was 88. The standard deviation in the math class was 3 and in the history class was 6.

a) Convert the math score to standard units.

b) Convert the history score to standard units.

c) Which score was higher with respect to the rest of the class?

3) Pam earned a 124 on his psychology midterm and an 87 on his foreign language midterm. The average score in accounting was a 102 with a standard deviation of 4.5. The average score in foreign language was 85 with a standard deviation of 2.

a) Convert the psychology score to standard units.

b) Convert the foreign language score to standard units.

c) Which score is higher with respect to the rest of the class?

4) Sal is on two bowling teams. On his first team, he scored a 212. This team had a team average of 242 with a standard deviation of 10. For his second team, Sal bowled a 197. This team averaged 174 with a standard deviation of 3.

a) Convert Sal's first score to standard units.

b) Convert Sal's second score to standard units.

c) Which score is higher with respect to the rest of the team?

Raw Score:

We can convert our formula for z score to a different formula that is helpful when we already know the z score but are looking for the __________________:

In many testing situations we hear the term raw score and z score. The raw score is just the score in the original measuring units, and the z score is the score in standard units.

Examples:

1) Troy took a standardized test to try to get credit for first-year Spanish by examination. If he got credit by exam, he would not need to take the courses. The standardized score was reported. His standardized score was 1.9. The mean score on the exam was 100 with standard deviation 12.

a) What was Troy's raw score?

b) The language department requires a raw score of 117 to get credit by examination for first-year Spanish. Will Troy get credit based on this exam?

2) Sam's z score on her college entrance exam is 1.7. If the raw scores have a mean of 364 and a standard deviation of 60 points, what is her raw score?

3) On a standardized test, Phil's z score is 1.75. If the raw scores have a mean of 364 and a standard deviation of 22 points, what is his raw score?

4) Amanda is a court reporter. She currently types 1.2 as a z score. If the raw scores of all court reporters across the nation average 222 with a standard deviation of 4, what is her raw score?

<u>Standard Units and Raw Scores:</u>

When looking at a range of scores, you should calculate the z score for both the upper limit and lower limit, and then set up an inequality to evaluate your data.

<u>Examples:</u>

1) In a class the final exam scores are normally distributed with a mean score of 82 and a standard deviation of 6. What percent of the exams are between 76 and 88?

2) In a class the final exam scores are distributed with a mean score of 85 and a standard deviation of 10 points. The B exams have scores ranging from 76 to 89. What are these scores in standard units? Indicate the possible z scores on a number line.

3) A professor gives A's to students in the class who have scores ranging from 91 to 99. The average score in the class is 88 with a standard deviation of 3. What are the z scores for the A students? Indicate the possible z scores on a number line.

4) Students in Dr. Z's class receive D's if they have grades of 66 to 74. The average score in the class is 90 with a standard deviation of 2. What are the z scores of the D students? Indicate the possible z scores on a number line.

5) Let x represent the life of a 60-watt light bulb. The x distribution has a mean of 1,000 hours with standard deviation of 75 hours. Convert each of the following x intervals into standard z intervals.

a) $450 \leq x \leq 1{,}350$ b) $900 \leq x \leq 1{,}100$ c) $990 \leq x \leq 1{,}010$

d) $500 \leq x$ e) $x \leq 300$ f) $x \leq 1{,}200$

6) Let x represent the average miles per gallon of gasoline that owners get from their new Nissan automobile. For this model the mean of the x distribution is advertised to be 44 mpg, with standard deviation of 6 mpg. Convert each of the following x intervals to standard z intervals.

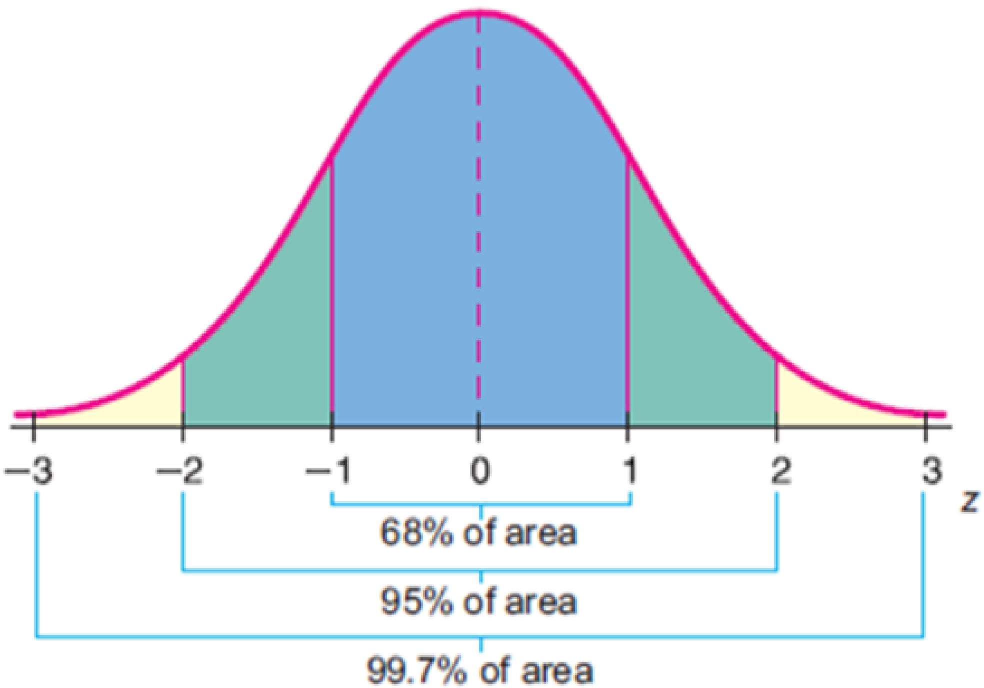
a) $x \geq 44$ b) $40 \leq x \leq 50$ c) $32 \leq x \leq 39$

7) A high school counselor was given the following z intervals concerning a vocational training aptitude test. The test scores had a mean of 450 points and a standard deviation of 35 points. Convert each x interval into an x test score interval.

a) $-1.14 \leq z \leq 2.27$ b) $z \leq -2.58$ c) $1.645 \leq z$

Areas Under the Standard Normal Curve:

The advantage of converting any normal distribution to the ________________ normal distribution is that there are extensive tables that show the area under the standard normal curve for almost any interval along the z-axis. The areas are important because they are equal to the probability that the measurement of an item selected at random falls in this interval. Thus the standard normal distribution can be useful.



We must know how to use this Table shown below. To do so take the highest value of z and break it down into two parts, the first part being the whole number, the decimal, and the tenths digit (ex. 2.9), and the second part being the hundredths digit (ex. 0.07). Now, just look it up on the chart using the first part in the vertical column and the second part in the horizontal column. Since the normal curve is symmetrical about its mean, we can use the table to find an area under the curve between a negative z value and 0 just in the same way we do positive values.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

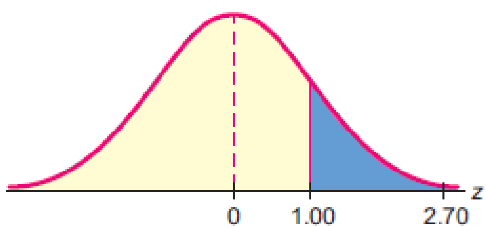
(a) Find the area between $z = 1.00$ and $z = 2.70$.

SOLUTION: First, sketch a diagram showing the area (see Figure 6-19). Because we are finding the area between two z values, we subtract corresponding table entries.

$$\begin{aligned}(\text{Area between } 1.00 \text{ and } 2.70) &= (\text{Area left of } 2.70) - (\text{Area left of } 1.00)\\ &= 0.9965 - 0.8413\\ &= 0.1552\end{aligned}$$

FIGURE 6-19

Area from $z = 1.00$ to $z = 2.70$



(b) Find the area to the right of $z = 0.94$.

SOLUTION: First, sketch the area to be found (see Figure 6-20).

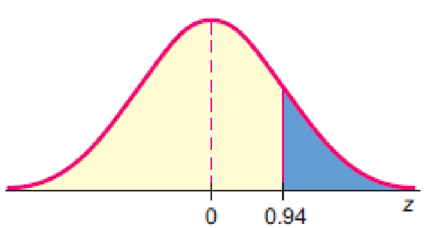
$$\begin{aligned}(\text{Area to right of } 0.94) &= (\text{Area under entire curve}) - (\text{Area to left of } 0.94)\\ &= 1.000 - 0.8264\\ &= 0.1736\end{aligned}$$

Alternatively,

$$\begin{aligned}(\text{Area to right of } 0.94) &= (\text{Area to left of } -0.94)\\ &= 0.1736\end{aligned}$$

FIGURE 6-20

Area to the Right of $z = 0.94$



Find the Areas Using the Table

To the right of $z = 0$

To the left of $z = -1.32$

To the left of $z = 0.45$

To the right of $z = 1.52$

To the right of $z = -1.22$

Between $z = 0$ and $z = 3.18$

Between $z = -2.18$ and $z = 1.34$

6.2 Homework

1) Convert each of the following to z-scores:

a) $\mu = 85$, $\sigma = 3.8$, $x = 92$

b) $\mu = 225$, $\sigma = 11.4$, $x = 208$

2) Convert each of the following to x-scores:

a) $\mu = 1100$, $\sigma = 40$, $z = -2.1$

a) $\mu = 920$, $\sigma = 55$, $x = 3.4$

3) Find the areas for the following using the table.

To the left of $z = 0$

To the left of $z = -0.47$

To the left of $z = 0.72$

To the right of $z = 0.15$

To the right of $z = -2.17$

Between $z = 0$ and $z = -1.93$

Between $z = -1.40$ and $z = 2.03$

6.3 Areas Under Any Normal Curve

Converting Normal Distributions to Standard Normal:

In many applied situations, the original normal curve is not the ______________________________________. Generally, there will not be a table of areas available for the original normal curve. This does not mean that we cannot find the probability that a measurement x will fall in an interval from a to b. What we must do is _______________ original measurements x, a, and b to z values.

NORMAL DISTRIBUTION PROBABILITY

Let x have a normal distribution with $\mu = 10$ and $\sigma = 2$. Find the probability that an x value selected at random from this distribution is between 11 and 14. In symbols, find $P(11 \le x \le 14)$.

SOLUTION: Since probabilities correspond to areas under the distribution curve, we want to find the area under the x curve above the interval from $x = 11$ to $x = 14$. To do so, we will convert the x values to standard z values and then use Table 5 of Appendix II to find the corresponding area under the standard curve.

We use the formula

$$z = \frac{x - \mu}{\sigma}$$

to convert the given x interval to a z interval.

$$z_1 = \frac{11 - 10}{2} = 0.50 \qquad \text{(Use } x = 11, \mu = 10, \sigma = 2.\text{)}$$

$$z_2 = \frac{14 - 10}{2} = 2.00 \qquad \text{(Use } x = 14, \mu = 10, \sigma = 2.\text{)}$$

The corresponding areas under the x and z curves are shown in Figure 6-23. From Figure 6-23 we see that

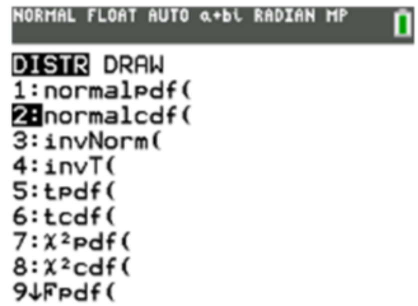
$$\begin{aligned} P(11 \le x \le 14) &= P(0.50 \le z \le 2.00) \\ &= P(z \le 2.00) - P(z \le 0.50) \\ &= 0.9772 - 0.6915 \qquad \text{(From Table 5, Appendix II)} \\ &= 0.2857 \end{aligned}$$

The probability is 0.2857 that an x value selected at random from a normal distribution with mean 10 and standard deviation 2 lies between 11 and 14.

Calculator Instructions

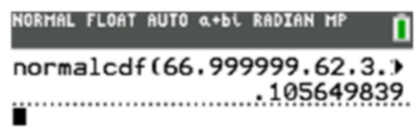
You can find the percent of a certain interval of the data by using your calculator.

1) 2ND VARS: Choose Option 2: normalcdf(



2) normalcdf(lower number, upper number, mean, standard deviation)

*If you aren't given an upper or a lower number, use 999999, or -999999



Examples:

In Problems 5–14, assume that x has a normal distribution with the specified mean and standard deviation. Find the indicated probabilities.

5. $P(3 \le x \le 6)$; $\mu = 4$; $\sigma = 2$
6. $P(10 \le x \le 26)$; $\mu = 15$; $\sigma = 4$
7. $P(50 \le x \le 70)$; $\mu = 40$; $\sigma = 15$
8. $P(7 \le x \le 9)$; $\mu = 5$; $\sigma = 1.2$
9. $P(8 \le x \le 12)$; $\mu = 15$; $\sigma = 3.2$
10. $P(40 \le x \le 47)$; $\mu = 50$; $\sigma = 15$
11. $P(x \ge 30)$; $\mu = 20$; $\sigma = 3.4$
12. $P(x \ge 120)$; $\mu = 100$; $\sigma = 15$
13. $P(x \ge 90)$; $\mu = 100$; $\sigma = 15$
14. $P(x \ge 2)$; $\mu = 3$; $\sigma = 0.25$

Inverse Normal Distribution

Sometimes we need to find z or x values that correspond to a given area under the normal curve. This situation arises when we want to specify a guarantee period such that a given percentage of the total products produced by a company last at least as long as the duration of the guarantee period. In such cases, we use the standard normal distribution table "in reverse." When we look up an area and find the corresponding z value, we are using the *inverse normal probability distribution.*

Inverse Normal: Use Table 5 of Appendix II to Find z Corresponding to a Given Area A $(0 < A < 1)$

(a) **Left-tail case:**
The given area A is to the left of z.

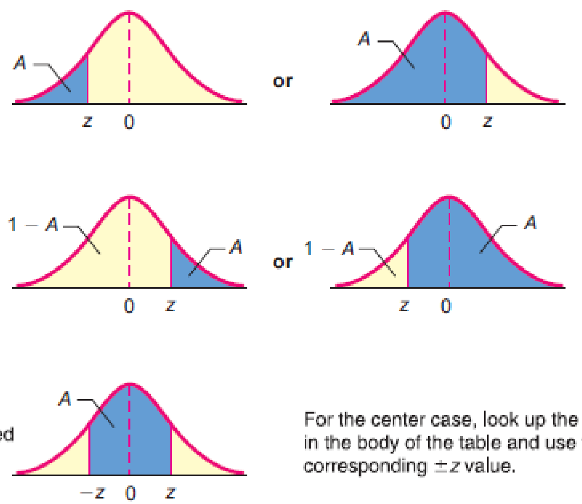
For the left-tail case, look up the number A in the body of the table and use the corresponding z value.

(b) **Right-tail case:**
The given area A is to the right of z.

For the right-tail case, look up the number $1 - A$ in the body of the table and use the corresponding z value.

(c) **Center case:**
The given area A is symmetric and centered above $z = 0$. Half of A lies to the left and half lies to the right of $z = 0$.

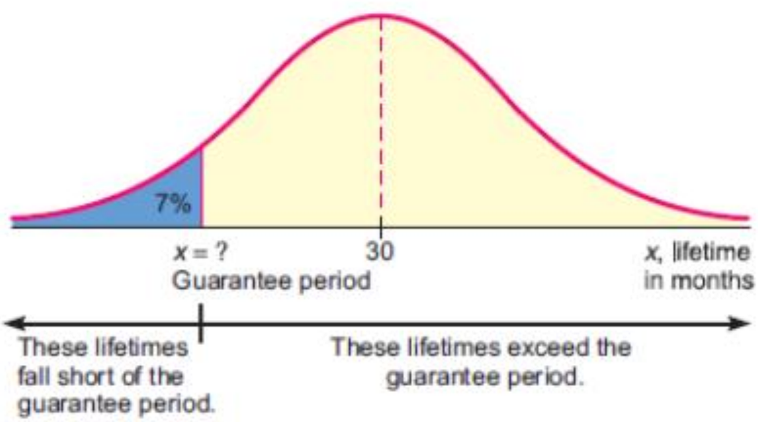
For the center case, look up the number $\frac{1-A}{2}$ in the body of the table and use the corresponding $\pm z$ value.



Left Tail Case

Magic Video Games, Inc., sells an expensive video games package. Because the package is so expensive, the company wants to advertise an impressive guarantee for the life expectancy of its computer control system. The guarantee policy will refund the full purchase price if the computer fails during the guarantee period. The research department has done tests which show that the mean life for the computer is 30 months, with standard deviation of 4 months. The computer life is normally distributed. How long can the guarantee period be if management does not want to refund the purchase price on more than 7% of the Magic Video packages?

SOLUTION: Let us look at the distribution of lifetimes for the computer control system, and shade the portion of the distribution in which the computer lasts fewer months than the guarantee period. (See Figure 6-26.)



z	.00	...	.07		.08	.09
:						
−1.4	.0808		.0708	↑	.0694	.0681
				0.0700		

We want to find the z value with 7% of the area under the standard normal curve to the left of z. Since we are given the area in a left tail, we can use Table 5 of Appendix II directly to find z. The area value is 0.0700. However, this area is not in our table, so we use the closest area, which is 0.0694, and the corresponding z value of $z = -1.48$ (see Table 6-5).

To translate this value back to an x value (in months), we use the formula

$$
\begin{aligned}
x &= z\sigma + \mu \\
&= -1.48(4) + 30 \qquad \text{(Use } \sigma = 4 \text{ months and } \mu = 30 \text{ months.)} \\
&= 24.08 \text{ months}
\end{aligned}
$$

The company can guarantee the Magic Video Games package for $x = 24$ months. For this guarantee period, they expect to refund the purchase price of no more than 7% of the video games packages.

Center Case:

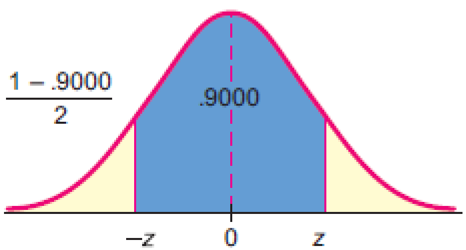
FIND z

Find the z value such that 90% of the area under the standard normal curve lies between $-z$ and z.

SOLUTION: Sketch a picture showing the described area (see Figure 6-28).

FIGURE 6-28

Area Between $-z$ and z Is 90%



We find the corresponding area in the left tail.

$$(\text{Area left of } -z) = \frac{1 - 0.9000}{2}$$
$$= 0.0500$$

Looking in Table 6-6, we see that 0.0500 lies exactly between areas 0.0495 and 0.0505. The halfway value between $z = -1.65$ and $z = -1.64$ is $z = -1.645$. Therefore, we conclude that 90% of the area under the standard normal curve lies between the z values -1.645 and 1.645.

TABLE 6-6 **Excerpt from Table 5 of Appendix II**

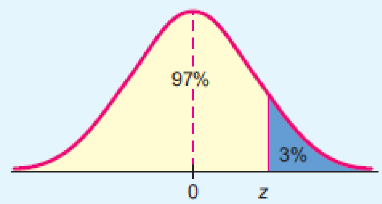
z	...	.04		.05
:				
−1.6		.0505	↑	.0495
			0.0500	

Right Tail Case

Find the z value such that 3% of the area under the standard normal curve lies to the right of z.

(a) Draw a sketch of the standard normal distribution showing the described area (Figure 6-29).

FIGURE 6-29 3% of Total Area Lies to the Right of z



(b) Find the area to the left of z.

Area to the left of $z = 1 - 0.0300 = 0.9700$.

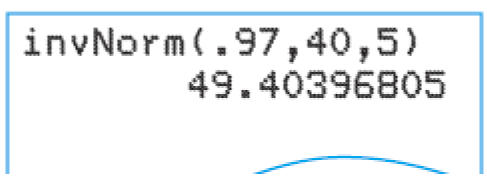
(c) Look up the area in Table 6-7 and find the corresponding z.

The closest area is 0.9699. This area is to the left of $z = 1.88$.

Calculator Instructions

functions for any normal distribution. For instance, to find an x value from a normal distribution with mean 40 and standard deviation 5 such that 97% of the area lies to the left of x, use the described instructions.

TI-84Plus/TI-83Plus Press the DISTR key and select **3:invNorm(area,μ,σ)**.



Examples

15. Find z such that 6% of the standard normal curve lies to the left of z.
16. Find z such that 5.2% of the standard normal curve lies to the left of z.
17. Find z such that 55% of the standard normal curve lies to the left of z.
18. Find z such that 97.5% of the standard normal curve lies to the left of z.
19. Find z such that 8% of the standard normal curve lies to the right of z.
20. Find z such that 5% of the standard normal curve lies to the right of z.
21. Find z such that 82% of the standard normal curve lies to the right of z.

6.3 Homework

1) Use your calculator to find the **PERCENT** of the data between the given values.

Given a mean of 84 and a standard deviation of 5.5, find:

a) P(between 81 and 90)

b) P(between 66.5 and 81)

c) P(less than 77)

d) P(more than 95)

2) Use your calculator to find the **DATA VALUE** for the given percents. *READ CAREFULLY

Given a mean of 850 and a standard deviation of 30, find:

a) the bottom 16%

b) the top 22%

c) the middle 40%

3) A person's blood glucose level and diabetes are closely related. Let x be a random variable measured in milligrams of glucose per deciliter (1/10 of a liter) of blood. After a 12-hour fast, the random variable x will have a distribution that is approximately normal with mean $\mu = 85$ and standard deviation $\sigma = 25$.What is the probability that, for an adult after a 12-hour fast,

(a) x is more than 60?

(b) x is less than 110?

(c) x is between 60 and 110?

(d) x is greater than 140 (borderline diabetes starts at 140)?

4) Accrotime is a manufacturer of quartz crystal watches. Accrotime researchers have shown that the watches have an average life of 28 months before certain electronic components deteriorate, causing the watch to become unreliable. The standard deviation of watch lifetimes is 5 months, and the distribution of lifetimes is normal.

(a) If Accrotime guarantees a full refund on any defective watch for 2 years after purchase, what percentage of total production will the company expect to replace?

(b) If Accrotime does not want to make refunds on more than 12% of the watches it makes, how long should the guarantee period be (to the nearest month)?

5) The resting heart rate for an adult horse should average about m _ 46 beats per minute with a (95% of data) range from 22 to 70 beats per minute. Let *x* be a random variable that represents the resting heart rate for an adult horse. Assume that *x* has a distribution that is approximately normal.

(a) Estimate the standard deviation of the *x* distribution.

(b) A horse whose resting heart rate is in the upper 10% of the probability distribution of heart rates may have a secondary infection or illness that needs to be treated. What is the heart rate corresponding to the upper 10% cutoff point of the probability distribution?

6.4 Normal Approximation to the Binomial Distribution

Normal approximation to the binomial distribution

Consider a binomial distribution where

n = number of trials

r = number of successes

p = probability of success on a single trial

$q = 1 - p$ = probability of failure on a single trial

If $np > 5$ and $nq > 5$, then r has a binomial distribution that is approximated by a normal distribution with

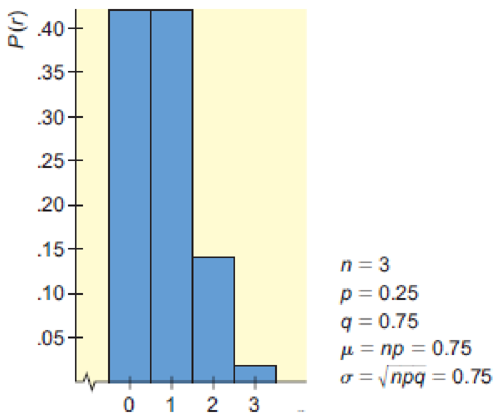
$$\mu = np \qquad \text{and} \qquad \sigma = \sqrt{npq}$$

Note: As n increases, the approximation becomes better.

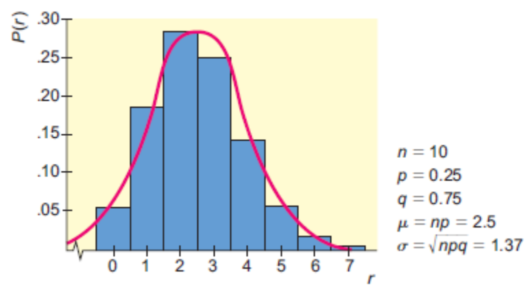
Binomial distribution graphs

Graph the binomial distributions for which $p = 0.25$, $q = 0.75$, and the number of trials is first $n = 3$, then $n = 10$, then $n = 25$, and finally $n = 50$.

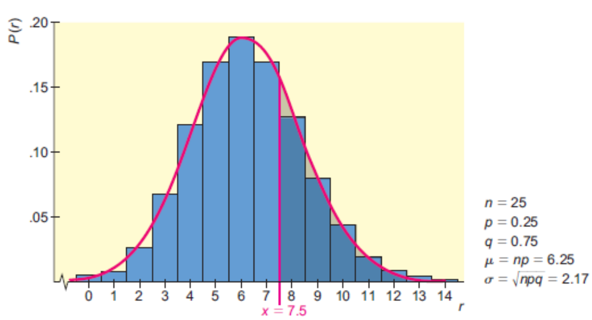
n = 3



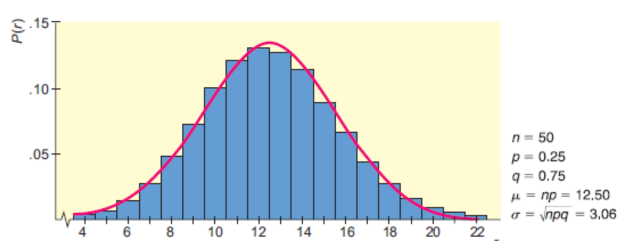
n = 10



n = 25



n = 50



Converting r Values to x Values

Remember that when using the normal distribution to approximate the binomial, we are computing the areas under bars. The bar over r goes from r – 0.5 to r + 0.5. If r is ______________ endpoint of an interval, we _____________________ to get the corresponding normal variable x. If r is a _______________ endpoint of an interval, we ___________________ to get the corresponding variable x.

The owner of a new apartment building must install 25 water heaters. From past experience in other apartment buildings, she knows that Quick Hot is a good brand. A Quick Hot heater is guaranteed for 5 years only, but from the owner's past experience, she knows that the probability it will last 10 years is 0.25.

(a) What is the probability that 8 or more of the 25 water heaters will last at least 10 years? Define success to mean a water heater lasts at least 10 years.

From many years of observation, a biologist knows that the probability is only 0.65 that any given Arctic tern will survive the migration from its summer nesting area to its winter feeding grounds. A random sample of 500 Arctic terns were banded at their summer nesting area. Use the normal approximation to the binomial and the following steps to find the probability that between 310 and 340 of the banded Arctic terns will survive the migration. Let r be the number of surviving terns.

(a) To approximate $P(310 \leq r \leq 340)$, we use the normal curve with $\mu =$ ________ and $\sigma =$ ________.

(b) $P(310 \leq r \leq 340)$ is approximately equal to $P($________ $\leq x \leq$ ________$)$, where x is a variable from the normal distribution described in part (a).

(c) Convert the condition $309.5 \leq x \leq 340.5$ to a condition in standard units.

Crime: Murder What are the chances that a person who is murdered actually knew the murderer? The answer to this question explains why a lot of police detective work begins with relatives and friends of the victim! About 64% of people who are murdered actually knew the person who committed the murder (*Chances: Risk and Odds in Everyday Life,* by James Burke). Suppose that a detective file in New Orleans has 63 current unsolved murders. What is the probability that

(a) at least 35 of the victims knew their murderers?

(b) at most 48 of the victims knew their murderers?

(c) fewer than 30 victims did *not* know their murderers?

(d) more than 20 victims did *not* know their murderers?

Grocery Stores: New Products The *Denver Post* stated that 80% of all new products introduced in grocery stores fail (are taken off the market) within 2 years. If a grocery store chain introduces 66 new products, what is the probability that within 2 years

(a) 47 or more fail?
(b) 58 or fewer fail?
(c) 15 or more succeed?
(d) fewer than 10 succeed?

6.4 Homework

1) A recent survey shows that 52% of American Households own an Xbox Game Console. A random sample of 85 students was looked at. What is the probability that at least 44 people own an Xbox?

2) It is estimated that 3.5% of the general population will live past their 90th birthday. In a graduating class of 753 high school seniors, what is the probability that:

(a) 15 or more will live beyond their 90th birthday?

(b) 30 or more will live beyond their 90th birthday?

(c) between 25 and 35 will live beyond their 90th birthday?

3) Ocean fishing for billfish is very popular in the Cozumel region of Mexico. In *World Record Game Fishes*, it was stated that in the Cozumel region about 44% of strikes on the line resulted in a catch. Suppose that on a given day a fleet of fishing boats got a total of 24 strikes. What is the probability that the number of fish caught was

(a) 12 or fewer?

(b) 5 or more?

(c) between 5 and 12?

4) More than a decade ago, high levels of lead in the blood put 88% of children at risk. A concerted effort was made to remove lead from the environment. Now, according to the *Third National Health and Nutrition Examination Survey* conducted by the Centers for Disease Control, only 9% of children in the United States are at risk of high blood-lead levels.

(a) In a random sample of 200 children taken more than a decade ago, what is the probability that 50 or more had high blood-lead levels?

(b) In a random sample of 200 children taken now, what is the probability that 50 or more have high blood-lead levels?